THE UNIVERSITY OF SYDNEY

Business School

Algebra

Mathematics Help Sheet

The University of Sydney Business School

Introduction

Terminology and Definitions

Integer	A whole number, as opposed to a fraction or a decimal, which can be positive or negative.
Constant	A constant is a number that is fixed. In algebraic expressions they are often called 'parameters'.
Variable	Also known as a pronumeral, a variable is an unknown value which can change depending on the value of constants or parameters. They are often denoted by letters such as x or y.
Co-efficient	A coefficient is a number (usually an integer) that appears in front of a variable as a multiple. For example, $5x$ represents $5 \times x$ or $x + x + x + x + x$.

Using algebraic expression

Variables can be added, subtracted, multiplied, and divided just like numbers. The same rules that apply in arithmetic generally also apply to algebraic expressions.

Addition and subtraction

When adding or subtracting variables, only **"like-terms"** can be added and subtracted. You can see in the examples below, only the x variables can be added with each other, and the y variables added together separately.

To add two "like-terms" simply add their coefficients, remembering that when no number appears in front of a variable, the coefficient is 1.

For example,

$$5x + x = 6x$$

$$2y + 3x + y = 3y + 3x$$

$$3x + 3y + 7x + 2y = 10x + 5y$$

Multiplication and Division

When multiplying variables, the same rules as multiplying numbers apply, except when you multiply two or more variables you will need to represent them as an exponential or power. The coefficients multiply as usual.

For example,

$$x \times x = x^2$$

$$x \times x \times x = x^3$$

$$7x \times 4 = 28x$$

$$x^3 \div x = x^2$$

$$\frac{6x^3}{2x} = 3x^2$$

In contrast to adding and subtracting variables, you do not have to treat like-terms as separate expressions. For example,

$$x \times x \times y \times y = x^2y^2$$

$$5x \times x \div 2y = \frac{5x^2}{2y}$$

Solving Simple Algebraic Equations

When a question asks you to solve for a variable such as, "Solve for the value of x", it means you have to find the value of x in terms of the parameters and other variables.

For example, solve the following equation for the value of x

$$5x = 10$$

Solution,

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

For solving more difficult algebraic equations, such as linear and quadratic functions, please see the "Solving Linear and Quadratic Functions" help sheet.

Factorisation and Expansion

Factorisation

If you have a long algebraic expression, you can simplify it by factorising. To do so, you need to reduce the expression into its highest common factor.

Consider the following expression,

$$2y + 8x + 6z$$

Each of the terms in the above expression share a common factor of 2. Thus you can divide each term by 2 and take that factor outside,

$$2(y + 4x + 3z)$$

Consider another example,

$$12xy - 6xz$$

This can be factorised into the following,

$$6x\ (2y - z)$$

Sometimes factorisation will involve more than one step.

For example,

$$5(3y + z) + 10x(3y + z)$$

Can be factorised into,

$$(5 + 10x)(3y + z)$$

However, notice that $(5 + 10x)$ can be factorised further into $5(1 + 2x)$. Hence the fully factorised form is,

$$5(1 + 2x)(3y + z)$$

Expansion

Expansion is simply the opposite of factorisation. If you have an algebraic expression in factorised form, you can expand it out by multiplying each expression within a bracket by each expression not within that bracket.

For example,

$$x(a + b) = ax + bx$$

$$2(y + z) = 2y + 2z$$

Consider when you have two brackets,

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(4+x)(5+y) = 20 + 4y + 5x + xy$$

Remember to multiply the negative sign when you have negative numbers. In the example below, the coefficient in front of x and y is -1 so you need to treat them as $-x$ and $-y$.

$$(2-x)(3-y) = 6 - 2y - 3x + xy$$

Notice when a bracket itself is squared you can separate it out,

$$(x+y)^2 = (x+y)(x+y)$$

$$(x+y)(x+y) = x^2 + xy + yx + y^2$$

Since xy is equivalent to yx, you can use the following rule,

$$(x+y)^2 = x^2 + 2xy + y^2$$

Function Notation

If you are trying to represent a relationship between two variables, may wish to express an algebraic equation as a function.

For example, you may wish to find an expression for the relationship between quantity of apples sold and revenue.

If you let apples be "a" and revenue be "R", "a" is what is called an **independent variable** and "R" is a **dependent variable**. This is because the value of "R" depends on the value of "a". If you sell apples for $2, then your function is this:

$$R = 2a$$

You could read the above as "revenue as a function of apples" and you could also write it as,

$$F(a) = 2a$$

Where $F(a)$ is **not** $F \times a$ but denoting "a function of a".

You can think of "a" as the input and $F(a)$ as the rule which tells you how to transform the a into an output.

For example, if $a = 30$,

$$F(30) = 2 \times 30 = 60$$

Consider another example,

$$F(x) = 8x + x^2 + 15$$

If $x = 5$,

$$F(5) = 40 + 25 + 15 = 80$$

Sigma Notation

Sigma notation allows you to represent long sums in a very concise manner. $\sum$ is the Greek capital letter Sigma and is also called the **summation operator**.

Consider the following expression,

$$x_1 + x_2 + x_3 + x_4 + x_5$$

You may come across this sort of expression in statistics. For example, you may collect data on students' grades and number of hours studied. You might assign x as the variable representing the number of hours someone has studied, thus x_1 is the number of hours that person 1 has studied and x_2 is the number of hours person 2 has studied, and so on. We call x_1 and x_2 observations.

Hence, if you have 5 observations and want to represent the "the sum of observations", you can write the expression using sigma notation as,

$$\sum_{i=1}^{5} x_i$$

The above expression is read as "the sum of x from 1 to 5. The i on the bottom of the operator tells you which observation to start from (here, it is 1), and the number on the top of the operator is the last observation to sum up.

You can use sigma notation to sum any numbers or variables. For example,

$$5 + 6 + 7 + 8 + 9 = \sum_{i=5}^{9} i$$

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \sum_{i=1}^{5} i^2$$

$$\sum_{i=1}^{6} 2i = 2 + 4 + 6 + 8 + 10 + 12$$

$$\sum_{i=1}^{n} x_1 = x_1 + x_2 + x_3 + \cdots + x_{n-1} + x_n$$